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Reply to the comment by Jacobs and Thorpe

We welcome the interesting comment by Jacobs and Thorpe (JT)[1], and are gratified that first-order rigidity is confirmed in their random bond model. We have worked with them to look again at the rigidity of Cayley trees, with the conclusion that the p_c on trees discussed our letter[2] is analogous to a spinodal point in thermodynamic transitions. In fact we have developed a new constraint equation, which when combined with our Cayley tree theory[3], leads to results which are numerically equivalent to the random bond model results presented in their comment. JT also argue that in two dimensions the "transition" is second order. We agree. There is a non-trivial diverging correlation length, so the "transition" is second order[2]. However, we raise the possibility in[2] that the "order parameter" has a first-order jump or a very small exponent. JT imply that first order rigidity is pathalogical. We *disagree*. We think that first-order rigidity is already well known experimentally. A common example is the collapse of a granular solid. Such a solid may be supported by internally sintered grains or by an external boundary. As these constraints are removed, the solid may eventually collapse in a first-order fashion (e.g. a landslide). This example is however a "directed" rigidity problem in that the central force constraints are primarily in compression. In detail our interpretation of the current data on trees and on "generic" triangular lattices is as follows:

(i) Within *Cayley tree models*, P_∞[2-4] and f'[1] both have jump discontinuities. The transition is then *first order*. The analysis of the random-bond problem by JT is for free boundaries, while our Cayley tree analysis[2,4] was for a rigid boundary. Motivated by the JT comment, we have analysed the tree model for a variety of boundaries. In addition we have found an expression for $f' = (-z/2g)(1-T_\infty^2)$ on trees[3]. The solution to the Cayley-tree equations, for $z = 6$, $g = 2$ and a rigid boundary gives $p_c = 0.605$[3]. However if we introduce a consistency equation[3] which removes boundary constraints, we find $p_c = 0.655$ (same as JT's random-bond model). In fact, in this case, our results for f' are equivalent to theirs. In the regime $0.605 < p < 0.655$, the solutions to the Cayley tree equations can be interpretted as being *metastable*. It is an open question as to whether this "spinodal" regime is accessable in granular solids.

(ii) On the *triangular lattice*, there is a diverging correlation length and a non-trivial correlation-length exponent[2,5,6]. Thus, we (and JT) have clearly demonstrated that the rigidity transition on the triangular lattice is *second order*[7], though whether it is "straightforward" or "conventional"[1] is a semantic issue[8]. On triangular lattices, we also both agree that the backbone goes to zero continuously with ($\beta' = 0.25 \pm 0.03$[2,6], $\beta' = 0.24 \pm 0.04$[5]). There is however a real difference in our interpretations of the data for the *infinite-cluster probability*, $P_\infty \sim (p-p_c)^\beta$. Taken alone, we think that the current numerical data on the triangular lattice is consistent with any $\beta < 0.175$. In particular, in our letter[2], we argued for a first order jump at p_c ($\beta \sim 0$) based primarily on the analysis of the "difference" or dangling ends. In contrast JT [5] analyse P_∞ itself and find ($\beta \sim 0.175 \pm 0.02$). There are very large finite-size effects in this problem, so that $L \sim 100,000$ is needed in order to find β precisely in two dimensions, both for the "difference"(see[1]) and for the infinite cluster itself. With careful use of memory this appears achievable in the next couple of years, using refinements of the current algorithms.

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[1] D. Jacobs and M.F. Thorpe, preceeding comment
[2] C. Moukarzel, P.M. Duxbury and P.L. Leath. Phys. Rev. Lett. **78**, 1480 (1997)
[3] P.M. Duxbury, D. Jacobs, M.F. Thorpe and C. Moukarzel preprint
[4] C. Moukarzel, P.M. Duxbury and P.L. Leath, Phys. Rev. **E55**, 5800 (1997)
[5] D. Jacobs and M.F. Thorpe, Phys. Rev. Lett. **75**, 4051 (1995); Phys. Rev. **E53**, 3682 (1996)
[6] C Moukarzel and P.M. Duxbury, Phys. Rev. Lett. **75**, 4055 (1995)
[7] It is also well established that the rigidity transition in two dimensions ($\nu = 1.16 \pm 0.03$)[2,6], $\nu = 1.21 \pm 0.06$[5]) is in a different universality class than connectivity percolation ($\nu = 4/3$)

[8] A jump discontinuity in an order parameter does occur in some "conventional" second order transitions - e.g. the helicity modulus of the 2-d xy model. The infinite cluster probability is like an order parameter of percolation problems.